

# IMPACT ON LOSS/RISK ASSESSMENTS OF INTER-MODEL VARIABILITY IN VULNERABILITY ANALYSIS

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*Fragility curves (FCs) constitute an emerging tool for the seismic risk assessment of all elements at risk. They express the probability of a structure being damaged beyond a specific damage state for a given seismic input motion parameter, incorporating the most important sources of uncertainties, i.e., seismic demand, capacity and definition of damage states. Nevertheless, the implementation of FCs in loss/risk assessments introduces other important sources of uncertainty, related to the usually limited knowledge about the elements at risk (e.g. inventory, typology). In this paper, it is developed a general methodology to merge into a single model the information provided by multiple FC models, weighting them according to their credibility/applicability. This combination enables to efficiently capture Inter-Model Variability (IMV) and to propagate it into risk/loss assessments, allowing the treatment of a large spectrum of vulnerability-related uncertainties, usually neglected. As case study, fragility curves for shallow tunnels in alluvial deposits, when subjected to transversal seismic loading, are developed with two conventional procedures, based on a quasi-static numerical approach. Noteworthy, loss/risk assessments resulting from such conventional methods show significant unexpected differences. Conventional fragilities are then combined in a Bayesian*

27 *framework, in which also probability values are treated as random variables, characterized by their*  
28 *probability density functions. The results show that this Bayesian Combined Model (BCM)*  
29 *efficiently projects the whole variability of input models into risk/loss estimations. This*  
30 *demonstrates that BCM is a suitable framework to treat IMV in vulnerability assessments, in a*  
31 *straightforward and explicit manner.*

32

33 ***1- INTRODUCTION***

34 In their original form, Fragility Curves (FCs) describe the probability of a structure being damaged  
35 beyond a specific damage state for various levels of ground shaking (e.g., ALA 2001; NIBS 2004).  
36 They are widely adopted in seismic expected loss and risk assessments, since they are a valuable  
37 tool to explicitly evaluate vulnerability of structures (e.g., NIBS 2004; Cornell and Krawinkler  
38 2000; Pitilakis et al 2006). Their applicability in estimating the probability of damage levels of  
39 particular element at risk contributes in the retrofitting decisions, emergency response planning and  
40 estimation of direct and indirect losses of built environments as well as lifeline systems (Pitilakis et  
41 al 2006; Kappos et al 2008; Azevedo et al 2010). Also, the use of fragility curves goes beyond the  
42 seismic risk analysis (e.g., Spence et al 2005), and it has been proposed as the general framework  
43 for vulnerability assessment in all natural risks (e.g., Douglas 2007; Shmidt et al 2011).

44 Many methods are used to generate FCs, based either on past recorded damages (e.g., Basöz  
45 et al 1999 for bridges; Maruyama et al 2010 for expressway embankments; Rossetto and Elnashai  
46 2003 for buildings), analytical modeling of the structures behavior under input ground motion (e.g.,  
47 Moschonas et al 2009 for bridges; Akkar et al 2005 for buildings), expert judgment (e.g., ATC-13;  
48 ATC-25), as well as, on a combination of such methods (hybrid methods, e.g., Kappos et al 2006 for  
49 buildings). All methods are based on the definition of a given set of damage states, possibly defined  
50 in terms of measurable quantities (Kappos 1997; Mackie and Stojadinovic 2003) and on an  
51 appropriate intensity measure (*IM*) describing ground motion (Pinto 2007; Mackie and Stojadinovic  
52 2003). Commonly, seismic FCs are represented as cumulative log-normal distribution (Shinozuka et

53 al 2000; Wen et al. 2003; Ellingwood and Kinali 2009, NIBS 2004). This assumption reduces the  
54 problem into the definition of the several parameters (e.g., Kennedy and Ravindra 1984; Choun and  
55 Elnashai 2010). Aleatory and epistemic uncertainties are often not separated and the problem is  
56 further reduced to the assessment of the two parameters of the log-normal distribution, that is,  
57 median  $m$  and logarithmic standard deviation  $\beta$ . Given damage data (real or modeled),  $m$  and  $\beta$  are  
58 assessed through different methodologies, from purely statistical methods, either classical (e.g.,  
59 Rossetto and Elnashai 2003; Shinozuka et al 2003) or Bayesian (e.g., Shinghal and Kiremidjian  
60 1996; Straub and Der Kiureghian 2008; Koutsourelakis 2010), to more physical assessment of  
61 critical points of structures, where seismic demand overcome structure capacity (e.g., Moschonas et  
62 al 2009; Nielson and DesRoches 2007). This procedure provides single best estimate FCs, which  
63 considers a composite variability parameter that does not explicitly separate out uncertainties (e.g.,  
64 Bhargava et al 2002).

65 All the steps toward the quantitative definition of FCs bring into the estimation procedures  
66 many uncertainties, both epistemic and aleatory. The most commonly analyzed uncertainty sources  
67 are the demand, capacity and damage state definition's uncertainties (e.g., NIBS 2004; Pinto 2007).  
68 Demand uncertainty reflects the fact that  $IM$  is not exactly sufficient, so different records of ground  
69 motion with equal  $IM$  may have different effects on the same structure (e.g. Karim and Yamazaki  
70 2001; Nielson and DesRoches 2007). Capacity uncertainty reflects the variability of structure  
71 properties as well as the fact that the modeling procedures are not perfect. Damage state definition  
72 uncertainties are due to the fact that the thresholds of the damage indexes or parameters used to  
73 define damage states are not known. Such uncertainties are usually assumed independent, while in  
74 many cases they are quantified through a single parameter based on expert judgement (e.g., NIBS  
75 2004).

76 The variety of analysis techniques, structural idealizations, seismic hazard and damage  
77 models being used, strongly influence the derived vulnerability curve shapes, and different choices  
78 have been seen to result in significant discrepancies between the seismic risk assessments made by

79 different authorities for the same location, structure type and seismicity (Rosseto and Elnashai  
80 2005). This inevitably leads to a large availability in literature of different FCs, even for similar or  
81 identical structures. As a matter of fact, even fragility curves derived from the same model or the  
82 same dataset may show important differences. As an example, Basöz and Kiremidjian (1998)  
83 developed different fragility curves for the same bridge damage data after the 1994, Northridge  
84 earthquake due to two different available sets of PGA values. Moreover, Shinozuka et al (2003)  
85 proposed fragility curves for the same dataset following another statistical analysis procedure. This  
86 significant variability has been explicitly shown in the ongoing European project SYNER-G (2010-  
87 2013), where different FCs, derived from different approaches, have been collected for several  
88 European typologies of buildings in a single tool. However, when FCs are applied in loss/risk  
89 assessments, in common practice only one single set of FCs is used (e.g. Kappos et al 2008;  
90 Azevedo et al 2010; Pitilakis et al 2010; Bommer et al 2008), usually referred to groups or  
91 typologies of structures. Often, if not always, there are no objective reasons to choose one set of  
92 curves instead of another, considering the large variety in FC , the variability of structures within  
93 each typology and the often inhomogeneous definition of typologies in different studies. In addition,  
94 the usually relatively poor knowledge about many or even most of the assets at risk further increases  
95 the (epistemic) uncertainty on the selection of one single set of FCs. However, the variability on the  
96 results due to different and subjective choices related to the vulnerability assessments is usually not  
97 considered at all, even if it may potentially introduce non predictable consequences in loss/risk  
98 assessments (e.g., Paté-Cornell 1996; Winkler 1996).

99 In seismic hazard, as well as in other fields, such Inter-Model Variability (IMV) is often  
100 assessed through Logic Trees, by mixing different approaches (Cornell and Merz 1975; McGuire  
101 1977; McGuire and Shedlock 1981; Giner et al 2002; Gruppo di Lavoro MPS 2004; SHARE  
102 project, 2009-2012) or fully treating all uncertainties (Kulkarni et al 1984; Coppersmith and Youngs  
103 1986; Electric Power Research Institute 1987; National Research Council 1988). In logic trees,  
104 alternative modeling choices are combined together, weighting each choice by its overall

105 applicability/credibility. The result is a discrete set of different hazard curves, which means that, for  
106 each single value of the *IM*, a discrete set of “possible” probabilities is assessed, often expressed as  
107 hazard maps at different percentiles (e.g., Gruppo di Lavoro MPS 2004). However, Logic Trees  
108 have several important drawbacks (e.g., Bommer and Scherbaum 2008). Among them, we mention  
109 (i) the fact that each branch duplication largely increases the computational effort, making  
110 practically difficult its application in medium/large scale loss/risk assessments (e.g., SYNER-G  
111 2010-2013); (ii) the difficulty in defining a mutually exclusive and collectively exhaustive set of  
112 alternative models and (iii) the consequent impossibility to treat uncertainty if only one model is  
113 available, even when weakly constrained or not completely applicable. In addition, since Logic  
114 Trees consider only a discrete number of alternative models, the whole variability among models is  
115 never explored. A more structured method to treat IMV consists of the use of the Bayesian  
116 probability concept, which allows us to assign a ‘subjective’ belief to different hypotheses, thing  
117 that is not conceivable in a classical framework (e.g., Lindley 1965; Gelman e al. 1995; Hofer  
118 1996). In practice, this means that future frequencies of events (i.e., their ‘probability’) can be  
119 treated as random variables, characterized by their own probability density functions (Gelman e al.  
120 1995). Such an idea has found lately applicability also in different fields of natural hazard analysis  
121 (e.g., Wen et al 2003; Marzocchi et al 2008, 2010; Grezio et al 2010).

122 In this paper, we propose an explorative application of the Bayesian probability concept to  
123 deal with IMV in vulnerability analysis in loss/risk assessments. In particular, we adopt a Bayesian  
124 framework to merge into a single overall prior model (Bayesian Combined Model, BCM) the  
125 information provided by different available methodologies, not necessarily homogeneous in their  
126 formulations, to be eventually fit to pertinent independent data, when available. Then, we evaluate  
127 the performance of BCM in propagating IMV into loss/risk assessments. Note that the goal of this is  
128 not to develop better or more refined FC models for one single asset, but to provide more reliable  
129 loss/risk assessments by propagating in them IMV. In fact, the epistemic uncertainty that emerges  
130 when FC models are selected in loss/risk assessment is relative to the applicability of FC, more than

131 to FCs by themselves, and it can be modeled accounting for the IMV. Indeed, this uncertainty is  
 132 usually significant when the vulnerability of a specific target area is modeled starting from generic  
 133 fragility models and it is not necessarily related to their structural analyses. In this, the presented  
 134 approach differs significantly from Bayesian FC approaches, since they have different goals (e.g.,  
 135 Straub and Der Kiureghian 2008; Koutsourelakis 2010).

136 In the followings, we first develop the Bayesian framework in which different and non-  
 137 homogeneous FC models can be combined in order to propagate IMV in loss/risk assessments  
 138 (*section 2*). The applicability of this procedure is then discussed (*section 3*), showing that it may  
 139 potentially treat a broad range of different sources of IMV in loss/risk assessments. Its practical  
 140 applicability is finally demonstrated and discussed through one realistic, though simplified, case  
 141 study (*section 4*).

142

## 143 **2- BAYESIAN COMBINATION MODEL (BCM) FOR VULNERABILITY ASSESSMENT**

144 The ultimate goal of all Fragility Curves (FC) models is to assess the probability that a given  
 145 damage state is reached or exceeded, given the occurrence of a certain level of the Intensity Measure  
 146 (IM) that describes the size of hazardous phenomena (e.g., ground shaking). The punctual  
 147 probability of each damage state ( $\pi_i$ ) can be obtained from the FCs, that is, for  $m$  damage states:

$$148 \quad \begin{cases} \pi_0 = 1 - F_1(IM) \\ \pi_1 = F_1(IM) - F_2(IM) \\ \pi_2 = F_2(IM) - F_3(IM) \\ \dots \\ \pi_m = F_m(IM) \end{cases} \quad (1)$$

149 where  $F_i(IM)$  indicates the FC (exceedance probability) relative to damage state  $i$ . In Eq. 1, the  
 150 index runs from 0 (no damages) to  $m$  (collapse) and, by definition,  $\pi_i > 0$  and sum to 1.

151 Given the occurrence of an earthquake, FC models and the assumed repair cost for each  
 152 damage state, it is possible to assess the expected losses for a given set of elements. In practice, the  
 153 expected cost to repair the generic  $k$ -th element at risk depends on its actual damage state, and can

154 be written as fraction of the total replacement cost ( $RC^{(k)}$ ):

155 
$$C_i^{(k)} = RC^{(k)} \cdot CDF_i \quad (2)$$

156 where the index  $i$  indicates the  $i$ -th damage state, and  $CDF_i$  (cost damage factor) is the fraction of  
157  $RC^{(k)}$  necessary to repair the  $i$ -th damage state. Note that, for simplicity, in this application,  
158 uncertainty in  $CDF_i$  is not included. However, also such uncertainties may be treated by sampling  
159 each  $CDF_i$  value from specific probability distributions (Stergiou and Kiremidjian 2006).

160 Assuming a seismic scenario, with a Monte Carlo (MC) simulation, it is possible to obtain a  
161 sample of losses for each element at risk (eq. 1), and thus a sample of the expected total losses, for  
162 the entire set of elements (eq. 2). Assuming that damages, given an IM scenario, are statistically  
163 independent, a single random damage state  $i^*$  can be selected for each element by comparing its  $\pi_i$   
164 with a random number in the interval  $[0,1]$ . Given the obtained random damage scenarios, the total  
165 loss for the whole set of elements, for each realization, can be evaluated by summing over all  
166 elements, that is

167 
$$l = \sum_k C_i^{(k)} \quad (3)$$

168 After repeating this procedure many times, we obtain a sample of possible total losses, given the IM  
169 values at each element location provided by a seismic scenario. From this sample of possible losses,  
170 we can evaluate the loss curve for a given scenario  $IM = im$ , defined as

171 
$$Lc(im; \vec{\pi}) = p(\geq l \mid im; \vec{\pi}) \quad (4)$$

172 in which its dependence on the fragility model's results (through  $\pi_i$ ) is explicitly reported. The  
173 average of expected losses  $l_m$  reads:

174 
$$l_m(a, \vec{\pi}) = \sum_k \sum_i C_i^{(k)} \pi_i \quad (5)$$

175 In risk assessments, losses for each  $im$  values are combined together. For example, in  
176 medium/large areas, the Average Annualized Earthquake Losses ( $AEL$ ) is often adopted as risk  
177 index (e.g., FEMA 2008).  $AEL$  can be assessed as the average of losses  $l_m(a, \pi_i)$  over all possible

178 ground motion values  $IM = im$ , that is

179 
$$AEL(\vec{\pi}) = \int_{IM} l_m(im, \vec{\pi}) dh(im) \quad (6)$$

180 where  $dh(im)$  is the non-cumulative hazard, for an exposure time of 1 year.

181 Different FC models for the same element at risk result in different assessments of the  
182 probabilities  $\pi_i$  in eq. 1, and consequently different punctual assessments of losses (eqs. 4 and 5)  
183 and risk (eq. 6). Here, we indicate with  $\pi_i^{(M_k)}$  the probability values resulting from the FC model  $M_k$ .  
184 With the goal of combining into a single high-level model the results of distinct standard FC  
185 models, without restriction in their formulation, as well as specific pertinent observations, the  
186 variability in  $\pi_i$  can be treated using the Bayesian probability concept, in which each probability  
187 value may be treated as a variable (e.g., Gelman et al. 1995). A schematic representation of this  
188 concept is reported in **Fig. 1**. It is beyond the goals of this paper to discuss the “philosophical”  
189 implications of this “probability of probability” assessment, but we stress that (i) this is a quite  
190 common procedure in dealing with this type of uncertainties, in many fields of science (starting  
191 from Mosimann 1962), including geophysics (e.g., Marzocchi et al. 2008, 2010; Grezio et al. 2010;  
192 Selva et al. 2010, 2012) and earthquake engineering (e.g., Paté-Cornell 1996; Wen et al 2003), and  
193 (ii) a similar philosophy is implicitly assumed whenever procedures like Logic Tree are adopted  
194 (e.g., Bommer and Scherbaum 2008 and references therein).

195 To assess the distribution  $[\vec{\pi}]$ , given a certain number of past data  $\{D\}$ , that is, a set of  
196 observed damage states due to a given IM value, we can make use of Bayes' rule, that is

197 
$$[\vec{\pi} | \{D\}] \propto [\{D\} | \vec{\pi}] [\vec{\pi}] \quad (7)$$

198 where  $[\vec{\pi}]$  is the prior,  $[\{D\} | \vec{\pi}]$  is the likelihood, and  $[\vec{\pi} | \{D\}]$  is the posterior probability  
199 distribution, the latter representing the final result of the inference. The choice of the functional  
200 form for prior and the likelihood distributions represents the core of the Bayesian inference and  
201 necessarily implies several assumptions about the modeled process (e.g., Gelman et al. 1995).

202 The much more common situation in loss/risk assessments is the scarcity or even non-



203 availability of  $\{D\}$  for most of the structures or classes of structures at risk in the target area. If large  
204 datasets were available, both the construction of specific fragility curves, and/or the selection of  
205 appropriate ones, would be a rather simple task, and thus IMV would be almost negligible. On the  
206 other hand, a rather high availability of theoretical models is quite common, at least for many  
207 structure typologies (e.g., SYNERG 2010-2013). However, often, there are not objective reasons to  
208 select one specific FC model, among the available ones. This uncertainty is not necessarily related  
209 to each FC model itself, but it is essentially linked to scarce knowledge about the target stock (see  
210 discussion in *section 3*). Hence, this epistemic uncertainty is related to the application of FC in  
211 loss/risk assessments, rather than to FCs by themselves, and it can be modeled accounting for the  
212 Inter-Model Variability (IMV). Since this variability essentially affects the prior distribution  $[\bar{\pi}]$ , we  
213 concentrate our attention on how IMV can be modeled at this level, to be fitted to pertinent  
214 independent (i.e., not used in the prior) data through Eq. 2, when these are available.

215 The specific goal is merging the information brought by a given number of starting models  
216 ( $M_k$ ,  $k=1,2, \dots, N_M$ ) into a single prior probabilistic model  $[\bar{\pi}]$  that accounts for the variability on  
217 their results, that is, the Inter-Model Variability (IMV), allowing the whole variability around the  
218 input model results to be explored. The distributions  $[\bar{\pi}]$  are, by definition, subjective (e.g., Gelman  
219 et al 1995). However, it is definitely more subjective to assume that only one of the available  
220 models is correct and/or applicable (Paté-Cornell 1996; Marzocchi et al. 2008). As a matter of fact,  
221 to assume one specific model means that the other FCs are assumed as wrong/non-applicable, even  
222 when they are almost equally acceptable. In addition, this assumption also implies that we do not  
223 distinguish at all between well-accepted and consolidated in literature models and less constrained  
224 ones. This may undoubtedly lead to uncontrolled biases in the final loss/risk assessments and to wrong  
225 conclusions/decisions (e.g., Paté-Cornell 1996; Woo 1999).

226 The first step to set BCM consists in assigning a specific functional form to the prior  
227 probability density function  $[\bar{\pi}]$  in Eq. 7. Given an IM value, the probabilities  $\pi_i$  for all damage

states represent a partition of the event ‘damage’. In other words, damage states form a set of exhaustive and mutually exclusive events, that is, for each model and each IM value, the punctual probability of damage states (e.g.,  $\pi_0$  for no damage,  $\pi_1$  for minor damages,  $\pi_2$  for moderate damages,  $\pi_3$  for extensive and  $\pi_4$  for complete damages) sum to 1. In this case, a common choice in statistics is an  $m$ -dimensional Dirichlet distribution (e.g., Mosimann 1962; Gelman et al. 1995; in natural hazards: Marzocchi et al. 2008, 2010; Selva et al. 2010, 2012):

$$[\vec{\pi}] = \text{Dir}_m(\vec{\pi}; \vec{a}) \quad (8)$$

where the vector  $[\vec{\pi}] = (\pi_0, \pi_1, \dots, \pi_m)$  contains all punctual probabilities, and the vector  $\vec{a} = (a_1, a_2, \dots, a_{m+1})$  contains the parameters of the Dirichlet distribution, that is, the hyper-parameters. Note that (i) the total variability of this prior distribution can be fully obtained by setting the hyper-parameters  $\vec{a}$  for all IM values, and that this is independent from the number of considered models, and (ii) the Dirichlet distribution automatically accounts for the correlation among the probabilities relative to the different damage states. Since the marginal distribution of the Dirichlet is a Beta distribution, which is unimodal, this functional choice implies the assumption that the transition among different FCs is expected to be soft, in other words, intermediate FCs are expected to exist and be applicable. The consequence of this on the applicability of BCM is discussed in *section 3*. The sum  $\sum_i a_i$  is inversely proportional to the total variance and thus represents a prompt of the global estimated IMV (e.g., Marzocchi et al. 2008).

The second step of BCM is to set the prior distribution, starting from the models results. Different procedures may be adopted, in which different levels of control of the average and/or the variance of the distribution  $[\vec{\pi}]$  are set. For example, the means of  $[\vec{\pi}]$  can be set as the (weighted) average of models’  $\vec{\pi}^{(Mk)}$ , and variance according to the (subjective) credibility of each model (e.g., Marzocchi et al. 2010). Here, we prefer a procedure in which both means and variance are controlled by the input model, since we want to investigate the whole IMV. In this case, the probability assessments of different input FC models can be treated as independent samples from

253 the unknown prior  $[\vec{\pi}]$ . Bayes' rule on  $\vec{a}$  reads:

$$254 \quad [\vec{a} | \{M\}] \propto [\{M\} | \vec{a}][\vec{a}] \quad (9)$$

255 where  $\{M\}$  stands for the models'  $\pi_i^{(Mk)}$ . Since in common practice the probabilities  $\pi_i$  are treated as  
 256 perfectly known, instead of adding a further layer to the Bayesian model, we prefer to keep the  
 257 model simple. Hence, we infer the best guess values of the hyper-parameters  $\vec{a}$ , starting from the  
 258 results obtained by the set of input models  $M_k$ . To assess the best guess  $a^*$ , we make use of a  
 259 Maximum A Posteriori (MAP) estimation, that is, we select the parameters that maximize  $[\vec{a} | \{M\}]$ .  
 260 The simplest choice for the prior  $[\vec{a}]$  is an improper non-informative uniform distribution, choice  
 261 that makes MAP equivalent to a standard Maximum Likelihood (ML) method. To consider models  
 262 with different credibility, the likelihood function  $[\{M\} | \vec{a}]$  can be weighted by the  
 263 credibility/applicability of each model (e.g., Wang et al. 2004; Ahmed et al. 2005). In this case, the  
 264 best guess hyper-parameters  $\vec{a}^*$  are selected by maximizing the weighted likelihood

$$265 \quad [\vec{\pi}] \equiv [\vec{\pi} | \{M\}] = Dir_m(\vec{\pi}; \vec{a}^*) \quad \leftarrow \quad \vec{a}^* = \operatorname{argmax}_a \left( \prod_k [Dir_m(\vec{\pi}^{(Mk)}; \vec{a})]^{w_k} \right) \quad (10)$$

266 where  $k$  runs over the models; the weight  $w_k$  represents the subjective credibility of the  $k$ -th model,  
 267 its actual values matter in a relative, more than absolute, sense;  $Dir_m(\vec{\pi}; \vec{a})$  represents the  $m$ -  
 268 dimensional Dirichlet probability density function with parameters  $\vec{a}$ ;  $\vec{\pi}^{(Mk)}$  is a vector containing  
 269 the guessed probabilities from the FC model  $M_k$  for a given value of IM (from Eq. 1). Both mean  
 270 and variance of  $[\vec{\pi} | \{M\}]$  are controlled by the input models. The underlying assumption is that  
 271 such input models well represent the whole IMV. In particular, since the variance of  $[\vec{\pi}]$  is  
 272 controlled by the models, it represents the *a posteriori* estimation of IMV, and it is small only when  
 273 the input model are in agreement. In addition, since  $\sum_i a^*_i$  changes at each IM level, the model  
 274 permits different levels of IMV to be considered.

275 The obtained prior  $[\vec{\pi}] \equiv [\vec{\pi} | \{M\}]$  can be input in Eq. 7, and updated in light of new pertinent  
 276 data, if any, in which case the IMV it will reshaped in agreement with new observations, that is

$$[\bar{\pi} | \{D\}, \{M\}] \propto [\{D\} | \bar{\pi}] [\bar{\pi} | \{M\}] = [\{D\} | \bar{\pi}] \cdot \text{Dir}_m(\bar{\pi}; \bar{a}^*, \bar{\pi}^{(Mk)}) \quad (11)$$

where a standard choice for the functional form of the likelihood  $[\{D\} | \bar{\pi}]$  is a Multinomial distribution (from Mosimann 1962). In the followings, to simplify the notations, we will always refer to the final result of BCM as  $[\bar{\pi}]$ , noted that this symbol may represent either the prior, or the posterior distribution.

It is worth to stress that, with this parameterization, peaks on specific probability values may arise only by a convergence of the input models, or by a large set of coherent observation  $\{D\}$ . Note also that BCM does not assume any functional form for FC models  $\{M\}$ , in order to extend its applicability to all non log-normal FC methodologies. Indeed, this is rather common both for seismic (e.g., Basöz and Kiremidjian 1998; Dueñas-Osorio et al. 2007) and non-seismic (e.g., Spence et al. 2005) vulnerability assessments. This possibility enables to make use as potential input models of the large set of studies available in literature, which is particularly important whenever uncertainty of epistemic type is treated (e.g., Marzocchi et al 2008, 2010).

The last step of BCM model is to propagate IMV in loss/risk assessment. Indeed, BCM models the variability in the probability assessments provided by different FC input models, which is the input for loss (eqs. 4 and 5) and risk (eq. 6) assessments. Hence, the IMV on  $\pi_i$  propagates in loss/risk assessments, providing variability in their numerical assessments. In other words, instead of single punctual assessments, BCM provides an estimate on the uncertainty on those values, since the probabilities  $\pi_i$  are not assumed as perfectly known.

In particular, for each single sample of  $\pi_i$ , different loss curves  $Lc$  can be evaluated through eq. 4. As a consequence,  $Lc$  will follow, at all IM levels, a probability density function  $[Lc]$ . The variability on  $Lc$  can be visualized by assessing expected losses at different levels of confidence, for all IM levels, that is:

$$Lc^{(x)}(im) \leftarrow p(\leq Lc^{(x)} | im; [\bar{\pi}]) = x \quad (12)$$

and the best guess estimation of the loss curve  $Lc^*$  can be obtained averaging over all possible  $\pi_i$ :

$$Lc^*(im) = \int_{\pi} Lc(im; \bar{\pi}) d[\bar{\pi}] d\pi_i \approx \sum_i Lc^{(x_i)}(x_i - x_{i-1}) \quad (13)$$

The approximation is valid for an adequate selection of percentiles  $x_i$  (e.g., Choun and Elnashai 2010). On the other hand, the variability on  $Lc^{(x)}$ , at different confidence level  $x$ , represents the variation induced by IMV in loss assessments.

As for the loss curve  $Lc$ , BCM estimates an entire distribution also for the mean loss assessment  $[l_m]$ . Also in this case, at each level of  $IM = im$ , we can define the mean loss at different level of confidence  $l_m^{(x)}$  as the quintiles of the distribution  $[l_m]$  as in Eq. 12, and the best guess value  $l_m^*$  as in eq. 13. As a consequence, this variability is transferred to  $AEL$  assessment. A proxy of the variability of  $AEL$  can be assessed obtained by assessing it with different levels of confidence on  $l_m$ , that is

$$AEL^{(x)} = \int_{IM} l_m^{(x)}(im) dh(im) \quad (14)$$

and, again, the best guess  $AEL$ , indicated as  $AEL^*$ , can be obtained as

$$AEL^* = \int_{\pi} AEL(\bar{\pi}) d[\bar{\pi}] \approx \sum AEL^{(x_i)}(x_i - x_{i-1}) \quad (15)$$

As for the loss curves  $Ls$ , also in this case  $AEL^{(x)}$ , when plotted as a function of  $x$ , shows how likely is it that a given  $AEL$  results an underestimation of the true one, and thus it represents a prompt of the variability induced in  $AEL$  by IMV. Similar considerations can be extended to all possible risk indexes.

In summary, BCM allows us to assess both best guess values and confidence on the estimation of losses and risk, propagating the IMV on vulnerability to the final results of loss/risk assessments. Noteworthy, such estimates have a lower likelihood of being biased than single models' results, since they account for more information (e.g., Woo 1999). On the other hand, the assessment of confidence on best guess values is of major importance (e.g., Paté-Cornell 1996), since it enables meaningful comparisons among losses/risks in different areas, as well as different losses/risks in the same area, in a multi-risk perspective (e.g., Grüntal et al 2006).

326 It is worth noting that BCM strongly differs from Bayesian inference procedures for fragility  
327 assessment proposed in literature (e.g., Shinghal and Kiremidjian 1996; Straub and Der Kiureghian  
328 2008; Koutsourelakis 2010), as well as from FCs evaluated at different confidence levels (without  
329 composite  $\beta$ -values, e.g., Kennedy and Ravindra 1984), where a distribution form is assumed (log-  
330 normal) and distributions' parameters are inferred in order to assess the 'best' curve for a given  
331 structure. On the opposite, BCM is targeted to produce more accurate loss/risk assessment by  
332 including IMV on FCs. Indeed, having different goals, BCM is not in alternative to such  
333 approaches, since they simply focus on different and complementary issues. This is clearly  
334 demonstrated by the fact that the results of one (or more) of these models may be input to BCM, by  
335 randomly drawing  $N$  probability assessments from the model  $\{\bar{\pi}^{(M)}\}$ , and use each sample as single  
336 estimation with weight  $w_i=w/N$  in Eq. 5, where  $w$  represents the weight of the overall model  $M$ . Of  
337 course, if large dataset of pertinent past data are available (same structures, large range of IMs), all  
338 models should lead to the same results, since IMV would be negligible in this case. Such data would  
339 enable us also to discriminate among different FC models, rejecting the ones that cannot 'explain'  
340 them (probabilities too far away from observed frequencies). This results also in BCM, since the  
341 application of Eq. 11 (Bayes' rule) with a large dataset would lead to posterior distributions highly  
342 peaked (very small variability) on the observed frequencies (Gelman et al 1995).

343

### 344 **3. APPLICABILITY OF BCM**

345 The uncertainty modeled by BCM essentially corresponds to the practical impossibility, common in  
346 many applications, to unequivocally select one specific FC model for a given structure (or typology  
347 of structures), because of the lack of background for one specific selection and the lack of resources  
348 to produce structure specific FCs (one for each element in the analyzed area).The presented  
349 procedure may be applied virtually to most of the sources of IMV, that is, whenever the selection of  
350 one specific FC model is highly disputable, for example:

351 1. FC models developed for similar configurations, but with different procedures that imply

significantly different results. For example, FCs obtained by different statistical procedures of the same empirical or numerical damage data, FCs obtained by different numerical procedures (e.g. dynamic or quasi-static analysis) or slightly different characteristics of the same structure, or FCs referred to the same structural class which were derived based on analytical or empirical procedures.

2. Lack of structure-specific FC models, leading to select nonspecific or generalized FC models from literature (e.g., FCs developed in different areas, with different construction practices)
3. FC models developed for slightly different input IMs, among which it is difficult to distinguish in long-term aggregated hazard assessments (e.g., different incidence angle of seismic waves for bridges)
4. Rough description of structures in the application area, leading to difficulty in classifying them into a well-defined taxonomy or, in the opposite, a rough taxonomy leading to broad classes (e.g., different number of floors in a generic typology of buildings)

Note that cases 1 and 2 are somehow different from cases 3 and 4. Indeed, in cases 1 and 2, one ‘true’ model equal for all elements is expected to exist, and thus  $\pi_i$  should be sampled at once for all identical elements. On the opposite, in cases 3 and 4, the variability is expected within the target stock, and the ‘true’ FC is expected to be different from element to element. Consequently, the probability  $\pi_i$  should be sampled independently for each element.

In *section 2*, we discussed that the choice of a Dirichlet distribution implies a rather soft transition in the set of applicable FCs, implying a limitation on the definition of broad/mixed typologies in the taxonomy. This limitation applies for the IMV described in cases 3 and 4, above, since only there, ‘different’ typologies are mixed up. For example, one typology can mix up structures with different number of floors, but cannot mix up masonry and RC structures, or RC structures designed with or without seismic code.

#### 378 4- CASE STUDY: LOSS/RISK ASSESSMENT FOR TUNNELS

379 To show the applicability of the BCM, a case study is considered. It has the goal of showing in  
380 details how BCM models IMV and it propagates this uncertainty into loss/risk assessments. To  
381 control all the parameters, we select a relatively simple application, that is, two input FC models  
382 and no past data. This configuration permits a simpler check of all steps, but any more complicated  
383 application do not introduce further either technical, or theoretical issues. This application is related  
384 to the IMV described in case 1 in *section 3*, that is, FCs obtained by different statistical approaches:  
385 due to the lack of adequate pertinent past data, these two approaches and the derived FCs can be  
386 considered to be equally applicable.

387 The final goal of this application is to assess the expected seismic losses and risk for a  
388 segment of bored tunnel (metro line) with a length of 1 km. Such a segment is assumed to be  
389 composed by 10 elements with a length of 100 m, each one of them laying in a specific soil type, as  
390 shown in **Fig. 2A**. The length of such segments is set so that the occurrence of damages in each  
391 element can be reasonably considered independent. The RC (repair cost) for each segment is set to  
392 0.5 million euro, while the value  $CDF_i$  (cost damage factor) for each damage state is reported in  
393 Table 1, col. 8, based on the repair model that is proposed by Werner et al (2006) for drilled tunnels  
394 in California. Such assumptions and values are indicative, but they are realistic for a preliminary  
395 application. For the application area, we consider the hazard curve in **Fig. 2B**, which is a reasonable  
396 hazard for the city of Thessaloniki, Greece (Pitilakis et al 2007). To concentrate on the effects of  
397 IMV in vulnerability assessment, we assume the hazard perfectly known, i.e., not affected by  
398 epistemic uncertainty.

399 Two different procedures to develop FCs for shallow tunnels in alluvial are then considered,  
400 based on the same modeling procedure. The vulnerability assessment is based on a quasi-static  
401 numerical analysis (Argyroudis and Pitilakis 2012), and the dataset of damages produced by this  
402 model are then used to estimate two different sets of log-normal FCs, through two quite common  
403 approaches, that is, linear regression method (M1, *appendix A*) and maximum likelihood method



(M2, *appendix A*). Such FC models represent a set of two equally acceptable procedures to derive FCs, and both could be independently selected to perform loss/risk assessments. Both procedures are repeated for two different tunnel typologies, differentiated by the soil conditions in which tunnels are built, i.e., soil C and D according to Eurocode 8 classification. All the obtained FC models (M1 and M2, for both typologies) consider Peak Ground Acceleration (PGA) as IM, and use 3 damage states (minor, moderate, and extensive-to-complete). The parameters are reported in **Table 1**.

The FCs of all models are reported in **Fig. 3**. We can note that M1 and M2 provide quite different results, in both soils. Given a PGA value, the punctual probabilities of the damage states ( $\pi_i$ , Eq. 1), as assessed by such models, are quite unlike, for both soils C and D. In *sections 4.1* and *4.2*, we will show that such differences lead to significantly different loss/risk estimations.

The results of these models are used to analyze the capability of BCM to combine and propagate IMV in loss/risk assessments. The analysis is divided in two parts. In *section 4.1*, we apply the BCM to one specific segment of tunnel built in soil C, in order to show how IMV propagates for one element and how different choices influence the results. In *section 4.2*, the preferred BCM model is applied to the schematic tunnel (metro line), analyzing the effect of IMV on the loss/risk assessments in a larger area, with different soil characterizations and with more than one element at risk.

#### **4.1- ONE ELEMENT: SINGLE TUNNEL ELEMENT IN SOIL C**

We first consider one single tunnel element in soil C. In **Fig. 4**, we report the loss/risk assessment results for each single model. In panels A1, we report the results of the loss assessment for one specific scenario, in this case set to  $\text{PGA} = 0.6 \text{ g}$ , in terms of the loss curve  $L_c$ , as assessed by the input models M1 and M2. The difference between the expected losses is significant, and it results in quite different probability estimations, being M1 results significantly larger than the corresponding values for M2. It is important to note that these considerations are not a specific

characteristic of the selected scenario: in panel A2,  $l_m(PGA)$  for all PGA values are reported as assessed by both models. In Fig. 4, panel A3, we report the risk index  $AEL$  for both models, considering the hazard curve in Fig. 2B.

To model IMV among such models, we first have to set their (subjective) credibility. To do so, we consider that models M1 and M2 have equal credibility, since based on the same data and on equally credible statistical procedures. Therefore, our best guess weighting scheme is  $w_1=0.5$ ,  $w_2=0.5$ . The sensitivity in this choice is then tested.

In **Fig. 5**, panels A1 to A3, we report the results of the loss/risk assessments for the best guess weighting scheme. In particular, we report best guess values and confidence intervals for all the assessments reported in Fig. 4, that is  $L_c$ ,  $l_m$  and  $AEL$ .  $AEL^{(x)}$  as obtained by model BCM is plotted as a function of  $x$ , indicating the confidence at which the true unknown  $AEL$  value is smaller than the various  $AEL$  values. For comparison, the punctual losses and risk index evaluated by M1 and M2, and best guess for BCM, are reported. Noteworthy, this variability in both loss and risk assessments cannot be dealt by variations of the  $\beta$ -value (e.g., Ferson and Ginzburg 1996) since, whatever  $\beta$ -value is used, any single choice provides only punctual probabilities (Eq. 1) and does not model the variability on such probabilities (eq. 1), and consequently cannot propagate it in risk/loss assessments (eqs. 2, 3 and 4).

In **Fig. 6**, we report the same results as above, with other weighting schemes. In particular, we select three further weighting schemes: 0.7 and 0.3; 0.9 and 0.1 and 0.1 and 0.9. The results are essentially the same, but here the distributions tend to move toward the model with greater weight. However, it is important to note that, also in this case, both mean values and confidence intervals still preserve memory of the less weighted model, and this memory tends to decrease for increasing difference on weights of models.

#### 4.2- MANY ELEMENTS AND TYPOLOGIES: SEGMENT OF METROLINE IN SOILS C & D

In this application, one seismic scenario consists of a PGA value for each one of the segments

456 and, for simplicity, (i) all the sites within the same soil type are assumed with equal PGA, and (ii)  
457 the PGA in soil type D is assumed equal to 1.3 times the PGA in soil type C. With these  
458 simplifications, the seismic scenario is completely defined by the selection of one PGA value for  
459 soil C. As exemplificative scenario, we select again a PGA value in soil C of 0.60 g. Also the  
460 comparison between BCM and ‘standard’ procedures results more complicated. Indeed, we have  
461 two models (M1 and M2) related to two typologies of tunnel (built in soil C and D), thus we must  
462 consider 4 possible combinations for standard FCs: *M1* in Soil C and *M1* in Soil D is indicated as  
463 M11; *M1* in soil C and *M2* in soil D as M12, and so on.

464 In **Fig. 7**, we report the same results that we reported in *section 4.2*, obtained by the best guess  
465 BCM model (equal weights) and compared with M1 and M2 results. In particular, the loss curve  $L_c$   
466 for a scenario of  $PGA = 0.6$  g (panel A1), the mean loss for all PGA levels (panel A2) and the risk  
467 index AEL (panel A3) are reported. Interestingly, the IMV is only very slightly reduced by staking a  
468 larger number of elements, and confidence intervals well describe the variability among the four  
469 possible combinations. In addition, in panel B, we report the distribution of losses for a scenario of  
470  $PGA = 0.6$  g.

471 Note that, to produce these results, an unknown unique ‘true’ model for each tunnel typology  
472 (soil) is assumed to exist, since all the elements of the same type are assumed identical and BCM  
473 variability represents alternative models for such a typology. In practice, this means that the  
474 distribution  $[\pi_i]$ , at each run of the model, it is sampled only once for all identical elements (i.e., in  
475 the same soil). As discussed in *section 3*, this is not always the case for all types of IMV.

476 To show the potentiality of BCM, we consider a further application, adding a third input FC  
477 model. As third FC model we consider the one proposed by ALA (2001) for alluvial (all soil types)  
478 tunnels with good construction. As first assumption, definition of minor and moderate damage states  
479 of M1/M2 is assumed equal to the one of M3. Since M3 does not include extensive-to-complete  
480 damages, this damage state is assumed not possible ( $\pi_3^{(M3)} \approx 0$ , i.e.,  $\pi_3^{(M3)} = \pi_3^{(M2)} \cdot 10^{-5}$ , for  
481 numerical reasons). Note that this addition implies an abrupt increase on the possible combinations

(i.e. eight: M111, M112, M121, M122, M211, M212, M221 and M222). On the contrary, with BCM this addition implies only the setting of a further weighting factor. In **Fig. 8**, we report the same results of Fig. 7, using as input the three models with weighting factors  $w_1=0.45$ ,  $w_2=0.45$ , and  $w_3=0.10$ . As expected, the distributions of losses and risk again well represent the input variability, and large tail toward smaller values of loss is present, since M3 estimations forecast significantly smaller losses.

488

### 489 **4.3- DISCUSSION OF RESULTS**

The results in Figs. 4 to 8 clearly show that the expected risk/losses are essentially ‘fragility model’-dependent. The loss curves  $L_c$  for a given scenario, as well as mean losses  $l_m$  and the risk index  $AEL$  show significant differences, when M1 and M2 are applied. For example, in Fig. 4, it is shown that, for one tunnel segment built in Soil C, the estimations of M2 are systematically greater than the M1 ones. Such differences are even more evident when the effects are stacked over a larger set of elements (Fig. 7). It is also evident that such differences are quite unreasonable, considering that M1 and M2 can be considered equally acceptable, but their results are highly incompatible. Note, for example, that the mean expected loss for M1, combination M11, results in the tail of the expected loss distribution of M2, combination M22 (see Fig. 7B). Such differences lead to the conclusion that at least one of the models M1 or M2 is significantly biased.

This apparent paradox is related to a lack in uncertainty evaluation, even though all the principal sources of uncertainty (demand, capacity and damage state definition) have been formally introduced in both M1 and M2 FCs. Interestingly, this uncertainty cannot be modeled simply by increasing the  $\beta$ -value, which formally describes only the error in the position of the medians  $m_i$ , since whatever parametrical choice is adopted, any single fragility cannot neither model nor propagate IMV in loss/risk assessments.

On the contrary, BCM allows us to describe and propagate the uncertainty related to the impossibility to choose among single FC models (IMV). As expected, BCM distributions generally

508 include all the values that either M1 and/or M2 produce, when equal applicability is assumed (Figs.  
509 5 and 7). The losses forecasted by BCM cover the whole variability previewed by both input models  
510 M1 and M2 (Fig. 7B), and do not simply average them. Of course, the frequency of each single loss  
511 depends on how likely it is for all input models (Figs. 7 and 8).

512 The most likely area for expected losses and risk lays between the ones of the input models,  
513 i.e., in the area in which all models provide likely values. In our opinion, this is highly reasonable,  
514 given the assumption that M1 and M2 are equally acceptable and likely (equal weights). In the Soil  
515 C case (Fig. 5), the best guess estimates of BCM are close to the mean of input models. On the  
516 opposite, the non-compatibility between FCs for soil D, moderate and extensive-to-complete  
517 damages (Fig. 3), leads to best guess estimates slightly shifted toward smaller values (Fig. 7A).

518 If equal acceptability is not assumed, BCM adapts its behavior to this information, provided  
519 by the models' weights. In practice, BCM's loss/risk estimates move toward the most likely model's  
520 ones, preserving in its variability memory of the less likely model's ones (Fig. 6, upper panels). This  
521 variability tends to disappear only when the difference in acceptability is rather high (Fig. 6, lower  
522 panel).

523 Noteworthy, the addition of further input models does not imply any supplementary neither  
524 theoretical nor computational effort, as it is demonstrated by the application in Fig. 8, where a third  
525 model is considered.

526

## 527 **5. FINAL REMARKS**

528 The choice of one single set of FCs is often largely subjective, and different fragility may lead to  
529 significantly different expected loss and risk assessments. Hence, this uncertainty, of epistemic  
530 type, strongly increases the possibility of biased loss/risk estimations and consequently weakens  
531 their practical usability (Fournier d'Albe 1979; Paté-Cornell 1996).

532 We have developed a Bayesian methodology (Section 2) that allows us to account and  
533 propagate into loss/risk assessments a large spectrum of uncertainties related to the application of

534 FC models in vulnerability assessments (Section 3), essentially linked to scarce knowledge about  
535 the target stock. This epistemic uncertainty is relative to the application of FC in loss/risk  
536 assessments, more than to FCs by themselves, and it can be modeled accounting for the Inter-Model  
537 Variability (IMV). This kind of variability cannot be treated by the standard uncertainty treatment  
538 (e.g., Ferson and Ginzburg 1996) and it is usually neglected. On the other hand, we have shown that  
539 the Bayesian Composition Model (BCM), explicitly modeling the variability in probability,  
540 appropriately and efficiently describes IMV by combining the results of different standard fragility  
541 analyses and pertinent data, explicitly quantifying the influence of such an uncertainty in loss/risk  
542 assessments. BCM considers, eventually with different weights, many inhomogeneous sources of  
543 information, independently from their formulation and their statistical representation. In addition,  
544 BCM does not involve a dramatic increase of the computation effort.

545       The quantification of IMV in loss/risk assessments is important, since it (i) significantly  
546 reduces the likelihood of biased cost/risk assessments, increasing their usability in practical  
547 applications, and (ii) explicitly assesses the confidence on loss/risk results. This permits meaningful  
548 and robust comparisons among losses/risks in different areas, as well as different losses/risks in the  
549 same area, in a multi-risk perspective (e.g., Grüntal et al 2006). Indeed, risk hierarchization is  
550 ultimately one of the most important goals of any loss/risk assessment. The probability that BCM  
551 results are biased is lower than the ones based on single models, since it is based on more  
552 information (e.g., Woo 1999). Obviously, to achieve the goal of an unbiased estimation in a real  
553 application, any pertinent information should be included, opportunely weighting models according  
554 to their different reliability/applicability.

555

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734

735 **Figure 1:** The Inter-Model Variability (IMV) is assessed on the punctual probability of the  $i$ -th  
736 damage state, i.e.,  $\pi_i$ , is represented through the probability density function  $[\pi_i]$ . In this case, as IM  
737 we used the Peak Ground Acceleration (PGA).  
738

739 **Figure 2.** *Panel A:* Hypothetical metro line with circular cross section and total length 1000m,  
740 passing through alluvial deposits of soil type C and D (EC8), divided in ten equal segments of  
741 100m. *Panel B:* Hazard curve for soil C.  
742

743 **Figure 3.** Set of fragility curves for circular tunnel following model  $M1$  and  $M2$ , (a) for soil C and  
744 (b) for soil D. The considered damage states are minor, moderate and extensive-to-complete.  
745

746 **Figure 4.**  $M1$  and  $M2$  loss/risk assessments. In particular, we report: in panel A1 the loss curve  $L_c$   
747 for a scenario  $PGA=0.6\text{ g}$ ; in panel A2 the mean loss curve  $l_m$ ; in panel A3, the risk index  $AEL$ .  
748

749 **Figure 5.** BCM loss/risk assessments with the best guess BCM model ( $w_1=w_2=0.5$ ), compared with  
750 input models  $M1$  and  $M2$ . In particular, we report in panel A1 the loss curve  $L_c$  for a scenario  
751  $PGA=0.6\text{ g}$  and in panel A2 the mean loss curve  $l_m$ . In here, the estimates for models  $M1$  and  $M2$   
752 are compared with BCM's best guess ( $L_c^*$  and  $l_m^*$ ) and confidence intervals. In panel A3, the risk  
753 index  $AEL$  for models  $M1$  and  $M2$  are compared with BCM's best guess  $AEL^*$  and  $AEL^{(x)}$ .  
754

755 **Figure 6:** BCM loss/risk assessments, compared with  $M1$  and  $M2$  assessments, for one tunnel  
756 element in soil C, with different weighting schemes: BCM with  $w_1=0.7$ ,  $w_2=0.3$  is reported in  
757 panels A; BCM with  $w_1=0.9$ ,  $w_2=0.1$  in panels B; BCM with  $w_1=0.1$ ,  $w_2=0.9$  in panels C. In all  
758 panels, as in Fig. 5, we report the results for (i) the loss curve  $L_c$  for a scenario  $PGA=0.6\text{ g}$ , (ii) the  
759 mean loss curve  $l_m$ , and (iii) the risk index  $AEL$ .  
760

761 **Figure 7:** Results of the loss/risk assessment for the metro line in Fig. 2. We report, as in Figs. 5 and  
762 6, the results for (i) the loss curve  $L_c$  for a scenario  $PGA=0.6\text{ g}$  in panel A1; the mean loss  $l_m$  as  
763 function of  $PGA$  in panel A2; the risk index  $AEL$  in panel A3. In panel B, it is reported the  
764 distribution of losses for the same scenario as for  $L_c$  ( $PGA=0.6\text{ g}$ ). For comparison, we report as  
765 vertical lines the average losses  $lm$  for  $M1$  and  $M2$  (configurations  $M11$  and  $M22$ ) for the scenario.  
766

767 **Figure 8:** Same as Fig. 7, but with the addition of  $M3$ . The weighting scheme is  $w_1=w_2=0.45$ ,  
768  $w_3=0.10$ .  
769

770 **Table 1.** Definition of damage states for the development of analytical fragility curves for tunnels  
 771 and estimated parameters of the fragility curves based on different methods

Damage State (ds <sub>i</sub> )	Range of damage index (DI)	Centr al value of DI	M1 - SOIL C		M2 - SOIL C		M1 - SOIL D		M2 - SOIL D		CDF
			<i>m<sub>i</sub></i> (g)	<i>β</i>	<i>m<sub>i</sub></i> (g)	<i>β</i>	<i>m<sub>i</sub></i> (g)	<i>β</i>	<i>m<sub>i</sub></i> (g)	<i>β</i>	
0. No damage	$M/M_{Rd} \leq 1.0$	-	-	-	-	-	-	-	-	-	0
1. Minor	$1.0 < M/M_{Rd} \leq 1.5$	1.25	0.55	0.70	0.52	0.55	0.47	0.75	0.41	0.60	0.10
2. Moderate	$1.5 < M/M_{Rd} \leq 2.5$	2.00	0.82		0.80		0.66		0.82		0.25
3. Extensive- to-Complete	$2.5 < M/M_{Rd} \leq 3.5$	3.00	1.05		1.39		0.83		1.91		0.75

772  
 773

## 774 **APPENDIX A: VULNERABILITY ASSESSMENT THROUGH FRAGILITY MODELS**

775 Recently, new analytical fragility curves for shallow metro tunnels have been proposed based on  
776 numerical simulation, considering both structural parameters, local soil conditions and variation of  
777 input ground motion (Argyroudis and Pitilakis, 2012). The quantification of the damage states is  
778 based on a damage index (DI) that is defined as the exceedance of strength capacity of the most  
779 critical sections of the tunnel (i.e. ratio of the developing moment (M) to the moment resistance  
780 (MRd) of the tunnel lining). The definition of damage states is then based on the range of damage  
781 index values (**Table 1, col. 1-3**). From the evaluated damage index, as a function of the *PGA* at the  
782 ground surface, the set of fragility curves relative to a discrete number of Damage States can be  
783 derived. Three different damage states are considered due to ground shaking: minor, moderate and  
784 extensive-to-complete damage ( $d_1$ ,  $d_2$ , and  $d_3$  respectively). Fragility curves (FC) are usually  
785 represented as a two-parameter (median and log-standard deviation) lognormal cumulative  
786 distribution functions. The development of FCs requires the definition of 4 parameters, 3 medians  
787  $m_i$  and 1 value of  $\beta$ , which are estimated in literature following different procedures.

788 Two procedures are adopted here: (i) a linear regression method (e.g. Nielson and  
789 DesRoches 2007; Pinto 2007), herein referred to as M1, and (ii) a maximum likelihood method  
790 (ML, e.g. Saxena et al. 2000; Shinozuka et al. 2000, 2003; Kim and Feng 2003; Straub and Der  
791 Kiureghian 2008), herein referred to as M2.

792 M1 has been recently published in Argyroudis and Pitilakis (2012). Such fragility functions  
793 are reported in **Table 1, col 4 5**, and plotted in **Figure 2 (light blue)** for the case of circular (bored)  
794 tunnel in soil type C and D. As regards M2, while ML is normally used starting from real data  
795 (Kalbfleish 1977), with the same philosophy it is here used with synthetic data produced by a  
796 model. In particular, as for M1, the starting database for M2 consists of the result of the coupled  
797 numerical analysis, i.e., the earthquake parameter (*PGA*) and the consequent damage index for the  
798 modeled tunnel ( $PGA_i$ ,  $DI_i$ ). By defining one threshold in *DI* for each damage state ( $t_1, t_2$ , and  $t_3$ ), the

799 data can be transformed as the result of a Bernoulli trial experiment, associating each PGA to the  
800 consequent expected damage state, i.e.,  $(PGA_i, y_i)$ , where  $y_i$  is equal to 1 or 0 depending on whether  
801 or not the tunnel section sustains the damage state, that is equal to 1 if it is observed the  $i$ -th damage  
802 state, 0 otherwise. To account for the uncertainty on damage state definition, for each starting datum  
803  $(PGA_i, DI_i)$ , a Monte Carlo simulation is performed, by producing  $N = 500$  couples of  $(PGA_i, y_i)$   
804 data, each one obtained by comparing out the observed value for the damage index  $(DI_i)$  with  
805 randomly sampled thresholds. The thresholds are sampled from uniform distributions in their  
806 confidence intervals (**Table 1, col. 2**). The fragility curves are assumed to be log-normally  
807 distributed, with different medians  $m_j$  and equal  $\beta$ -value. The best guess values for the parameters  
808  $(m_i'$  and  $\beta')$  are obtained by numerically maximizing, as a function of  $m_j$  and  $\beta$ , the likelihood  
809 function  $L$ . The obtained values  $(m'$  and  $\beta')$  account for the demand uncertainty, since different  
810 seismic records as input for the coupled numerical analysis are used, and the damage state definition  
811 uncertainty by randomly selecting the DI thresholds (Eq. A.2). Among the principal sources of  
812 uncertainties, only the capacity uncertainty is not yet considered. Thus, it is added to the results of  
813 the analysis as the square root of the sum of squares of  $\beta'$  and 0.3 (e.g., NIBS 2004).

814 The obtained  $\beta''$ -value, which includes also capacity uncertainty, is then put into the  
815 likelihood function that, this time, is a function of the medians  $m_j$  only. The best guess medians  
816  $(m_i'')$  are obtained by numerically maximizing  $\ln(L')$  and, together with the total  $\beta''$ -value, represent  
817 the best guess parameters for the log-normal distribution. From this analysis, we obtain the final  
818 parameters for M2, as reported in **Table 1, col. 6-7** and plotted in **Figure 2 (dark blue)**.